磁化プラズマ中におけるミリ波光渦の伝播に関する有限差分時間領域法（FDTD）解析

FDTD Analysis of Propagation Characteristics of Millimeter Wave Vortex in Plasma

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Motivation

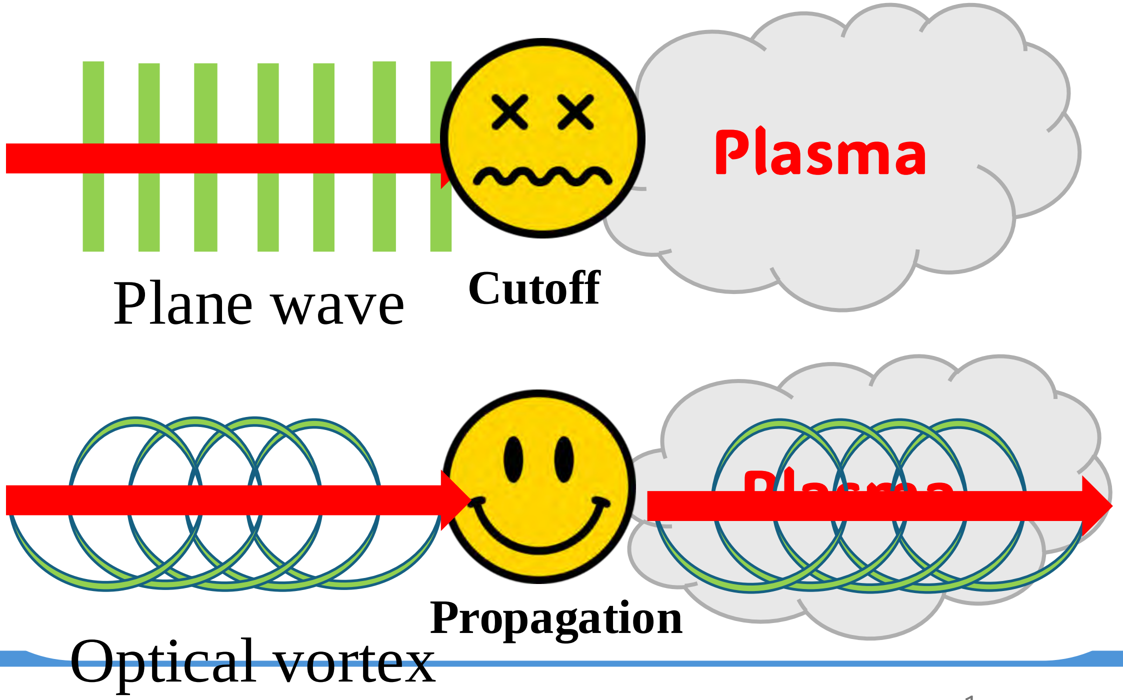
- To achieve fusion condition, **high temperature (> 108 K~104 keV) plasma** is essential
- Especially in the initial stage, external energy input is required to heat the plasma to ignition conditions

→ Electron Cyclotron Heating (E

Plasma Heating by millimeter-wave vortex

Core Challenges in ECH

- High-density plasma heating is limited
- Conventional ECH (plane wave) canno

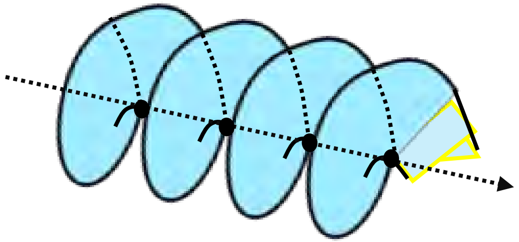


Explore the optical vortex propagation in magnetized plasma and its potential for functional plasma heating by FDTD

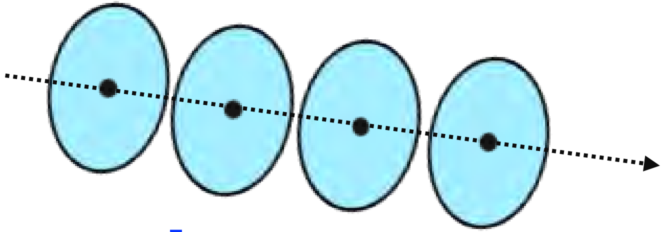
*FDTD=Finite-Difference Time Domain

T. Tsujimura and S. Kubo, Physics of Plasmas, 28, 012502 (2021)

Optical Vortex



Optical vortex

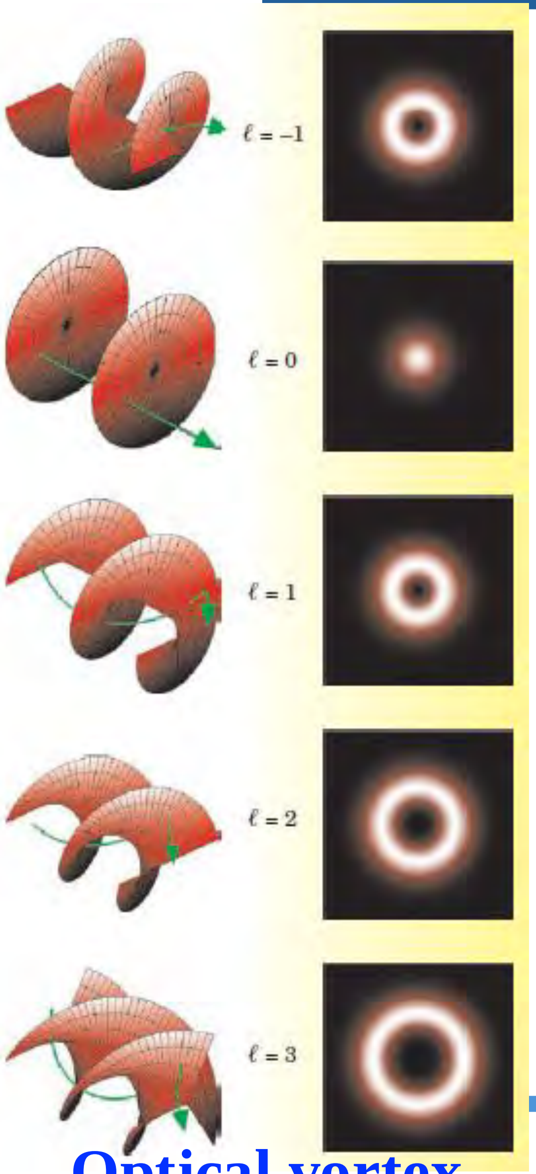


Plane wave

- **Helical wavefront**
- **Zero intensity in the center**
- **Phase varies azimuthally:** $e^{il\emptyset}$ **(topological charge** l**)**

  Optical vortex: $e^{il\emptyset}$ e^{ikz}

  Plane wave: e^{ikz}
- **orbital angular momentum (OAM) in addition to spin angular momentum (SAM)**



l=-1

l=0

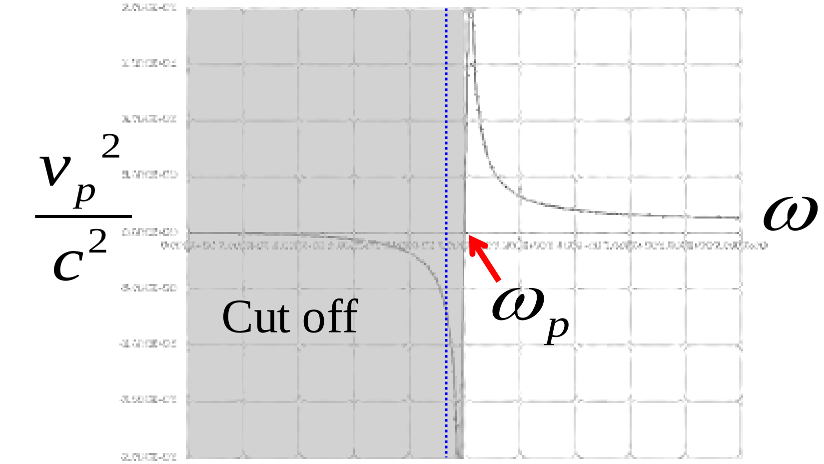
l=1

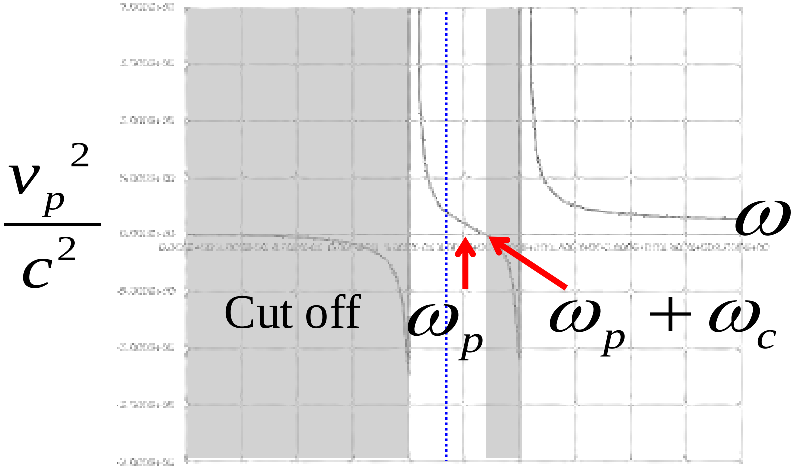
l=2

l=3

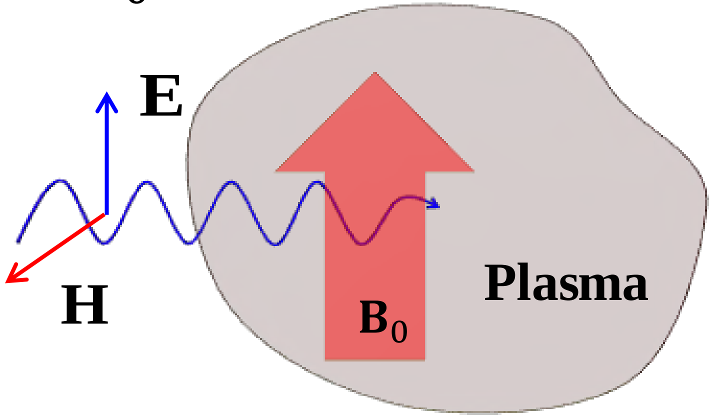
Optical vortex

Wave propagation mode in magnetized cold-plasma

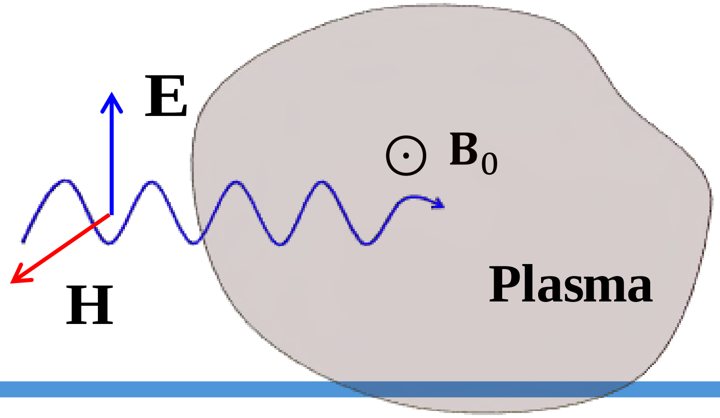




B_0//E (O mode)



$B_0 \perp$ E (X mode)



In this research:

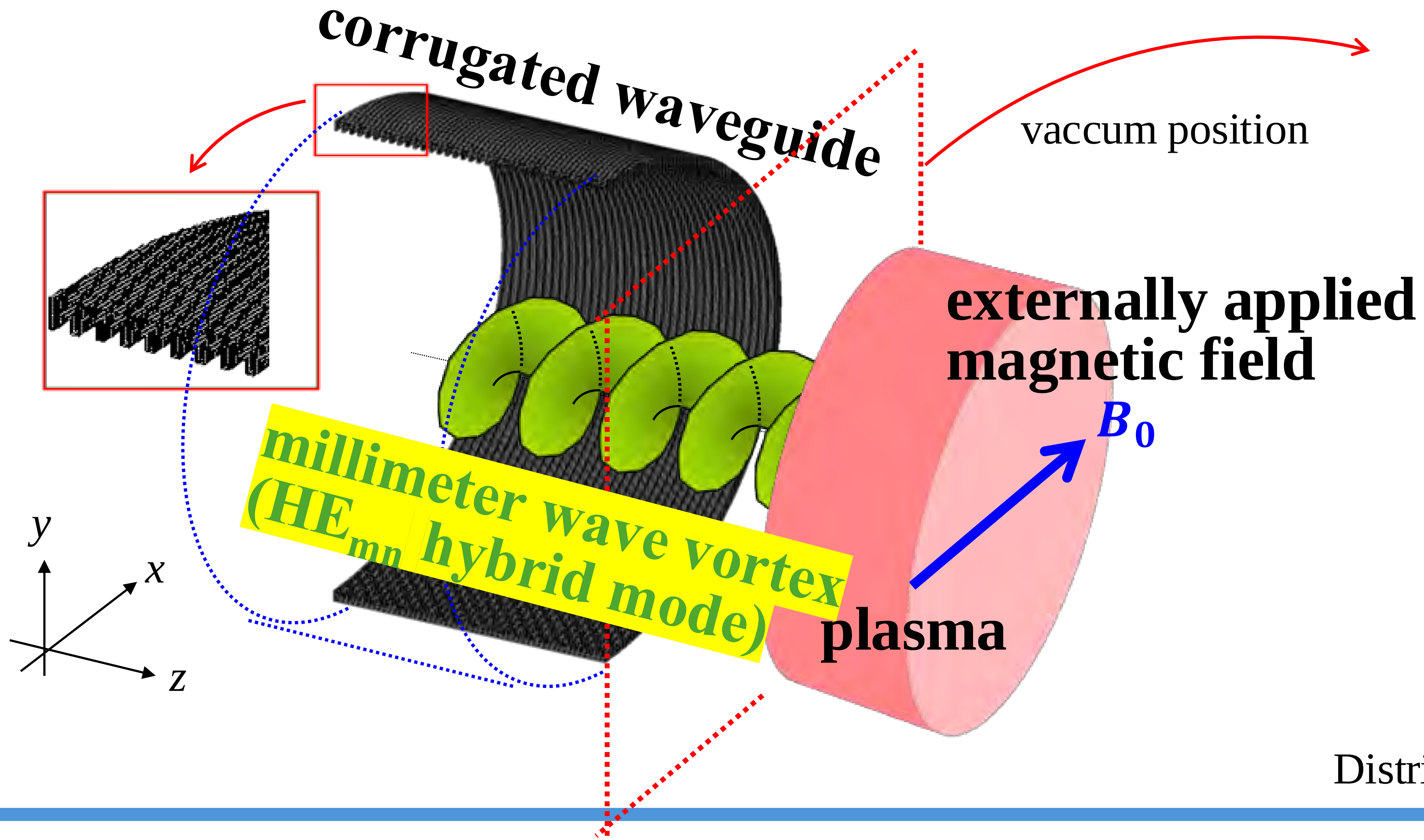
Plane wave (f =77GHz)

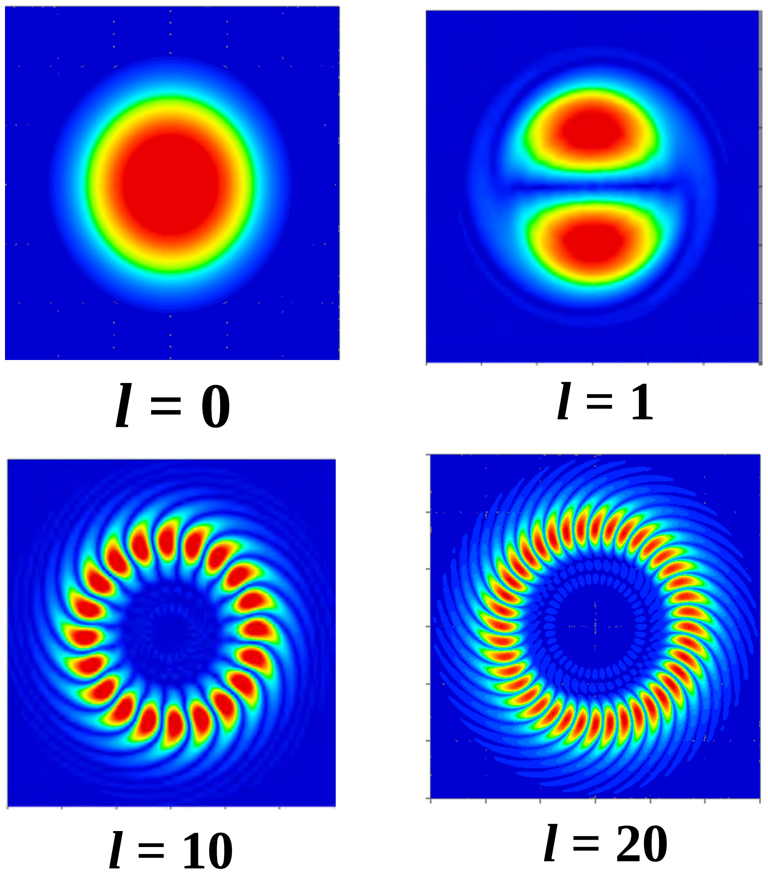
- **Linearly polarized (O-mode)**
- **$f < f_p$ (f_p≈94 GHz)**

→ Cut off

What will happen under same frequency for Optical Vortex ?

The Whole configuration of plasma heating by optical vortex





Distribution of electric field intensity (x-y)

1. C. Wang et al. JJAP, Vol.63, No.9, 09SP08 (5 pages), Sept. 2024.

The modeling of magnetized plasma

Magnetized plasma (Drude-Lorentz model) FDTD:Finite-difference time domain

$$\frac{d\boldsymbol{J}}{dt}+\nu\boldsymbol{J}+\omega_0{}^2\boldsymbol{P}=\varepsilon_0\omega_p{}^2\left(\boldsymbol{E}+\frac{\boldsymbol{J}}{en_e}\times\boldsymbol{B}\right)\xrightarrow{\text{FDTD}}\frac{\mathbf{J}^{n+\frac{1}{2}}-\mathbf{J}^{n-\frac{1}{2}}}{\Delta t}+\nu\frac{\mathbf{J}^{n+\frac{1}{2}}+\mathbf{J}^{n-\frac{1}{2}}}{2}+\omega_0^2\mathbf{P}^n=\varepsilon_0\omega_p^2\mathbf{E}^n+\frac{\varepsilon_0\omega_p^2}{en_e}\frac{\mathbf{J}^{n+\frac{1}{2}}+\mathbf{J}^{n-\frac{1}{2}}}{2}\times\mathbf{B}_0$$

$$\boldsymbol{J}=\frac{d\boldsymbol{P}}{dt}\xrightarrow{\text{FDTD}}\frac{\mathbf{P}^{n+1}-\mathbf{P}^n}{\Delta t}=\mathbf{J}^{n+\frac{1}{2}}$$

Millimeter-wave (Maxwell equations)

$$\nabla\times\mathbf{E}=-\mu\frac{\partial\mathbf{H}}{\partial t}\xrightarrow{\text{FDTD}}\mathbf{H}^{n+\frac{1}{2}}=\mathbf{H}^{n-\frac{1}{2}}-\frac{\Delta t}{\mu}\nabla\times\mathbf{E}^n$$

$$\nabla\times\mathbf{H}=\mathbf{J}+\sigma\mathbf{E}+\varepsilon\frac{\partial\mathbf{E}}{\partial t}\xrightarrow{\text{FDTD}}\mathbf{E}^{n+1}=\frac{\frac{\varepsilon}{\Delta t}-\frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t}+\frac{\sigma}{2}}\mathbf{E}^n+\frac{1}{\frac{\varepsilon}{\Delta t}+\frac{\sigma}{2}}\nabla\times\mathbf{H}^{n+\frac{1}{2}}-\frac{1}{\frac{\varepsilon}{\Delta t}+\frac{\sigma}{2}}\mathbf{J}^{n+\frac{1}{2}}$$

$\omega_0 = 0$, $\nu = 0$, 77 GHz, $B_{0\,x} = 2$ T, $n_e = 1.103 \times 10^{20}$

Frequency domain:

$$\frac{d\boldsymbol{J}}{dt}+\nu\boldsymbol{J}+\omega_0{}^2\boldsymbol{P}=\varepsilon_0\omega_p{}^2\left(\boldsymbol{E}+\frac{\boldsymbol{J}}{en_e}\times\boldsymbol{B}\right)$$

$$\downarrow$$

$$i\omega\boldsymbol{J}+\nu\boldsymbol{J}+\frac{\omega_0{}^2}{i\omega}\boldsymbol{J}-\frac{e}{m}\boldsymbol{J}\times\boldsymbol{B}_0=\varepsilon_0\omega_p{}^2\boldsymbol{E}$$

$$\boldsymbol{\sigma}=\begin{pmatrix}-\frac{i\varepsilon_0\omega_p^2}{\omega} & 0 & 0\\ 0 & -\frac{i\varepsilon_0\omega_p^2\,\omega}{\omega^2-\Omega_x^2} & -\frac{\varepsilon_0\omega_p^2\,\Omega_x}{\omega^2-\Omega_x^2}\\ 0 & \frac{\varepsilon_0\omega_p^2\,\Omega_x}{\omega^2-\Omega_x^2} & -\frac{i\varepsilon_0\omega_p^2\,\omega}{\omega^2-\Omega_x^2}\end{pmatrix}$$

1. C. Wang et al. JJAP, Vol.63, No.9, 09SP08 (5 pages), Sept. 2024.

FDTD formulation of electromagnetic waves in plasma

$$\boldsymbol{H}^{n+\frac{1}{2}} = \boldsymbol{H}^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\nabla \times \boldsymbol{E}^{n}$$

$$H_x^{n+\frac{1}{2}} = H_x^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\left(\frac{\partial E_z^n}{\partial y} - \frac{\partial E_y^n}{\partial z}\right)$$

$$H_y^{n+\frac{1}{2}} = H_y^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x}\right)$$

$$H_z^{n+\frac{1}{2}} = H_z^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\left(\frac{\partial E_y^n}{\partial x} - \frac{\partial E_x^n}{\partial y}\right)$$

$$\boldsymbol{E}^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\boldsymbol{E}^{n} + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\nabla \times \boldsymbol{H}^{n+\frac{1}{2}} - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\boldsymbol{J}^{n+\frac{1}{2}}$$

$$E_x^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}E_x^n + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\left(\frac{\partial H_z^{n+\frac{1}{2}}}{\partial y} - \frac{\partial H_y^{n+\frac{1}{2}}}{\partial z}\right) - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}J_x^{n+\frac{1}{2}}$$

$$E_y^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}E_y^n + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\left(\frac{\partial H_x^{n+\frac{1}{2}}}{\partial z} - \frac{\partial H_z^{n+\frac{1}{2}}}{\partial x}\right) - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}J_y^{n+\frac{1}{2}}$$

$$E_z^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}E_z^n + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\left(\frac{\partial H_y^{n+\frac{1}{2}}}{\partial x} - \frac{\partial H_x^{n+\frac{1}{2}}}{\partial y}\right) - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}J_z^{n+\frac{1}{2}}$$

Millimeter-wave Maxwell's equation

$$\nabla \times \mathbf{E} = -\mu\frac{\partial \mathbf{H}}{\partial t} \xrightarrow{\text{FDTD}} \mathbf{H}^{n+\frac{1}{2}} = \mathbf{H}^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\nabla \times \mathbf{E}^{n}$$

$$\nabla \times \mathbf{H} = \mathbf{J} + \sigma\mathbf{E} + \varepsilon\frac{\partial \mathbf{E}}{\partial t} \xrightarrow{\text{FDTD}} \mathbf{E}^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\mathbf{E}^{n} + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\nabla \times \mathbf{H}^{n+\frac{1}{2}} - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\mathbf{J}^{n+\frac{1}{2}}$$

FDTD formulation of electromagnetic waves in plasma

$$\begin{pmatrix} \frac{1}{\Delta t}+\frac{\nu}{2} & -\frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{z0}}{2} & \frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{y0}}{2} \\ \frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{y0}}{2} & \frac{1}{\Delta t}+\frac{\nu}{2} & -\frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{z0}}{2} \\ -\frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{z0}}{2} & \frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{y0}}{2} & \frac{1}{\Delta t}+\frac{\nu}{2} \end{pmatrix} \begin{pmatrix} J_x^{n+\frac{1}{2}} \\ J_y^{n+\frac{1}{2}} \\ J_z^{n+\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} \left(-\frac{1}{\Delta t}+\frac{\nu}{2}\right)J_x^{n-\frac{1}{2}} + \frac{\varepsilon_0\omega_p^2}{2en_e}\left(J_y^{n-\frac{1}{2}}B_{z0} - J_z^{n-\frac{1}{2}}B_{y0}\right) + \varepsilon_0\omega_p^2 E_x^{\,n} - \omega_0^2 P_x^{\,n} \\ \left(-\frac{1}{\Delta t}+\frac{\nu}{2}\right)J_y^{n-\frac{1}{2}} + \frac{\varepsilon_0\omega_p^2}{2en_e}\left(J_z^{n-\frac{1}{2}}B_{x0} - J_x^{n-\frac{1}{2}}B_{z0}\right) + \varepsilon_0\omega_p^2 E_y^{\,n} - \omega_0^2 P_y^{\,n} \\ \left(-\frac{1}{\Delta t}+\frac{\nu}{2}\right)J_z^{n-\frac{1}{2}} + \frac{\varepsilon_0\omega_p^2}{2en_e}\left(J_x^{n-\frac{1}{2}}B_{y0} - J_y^{n-\frac{1}{2}}B_{x0}\right) + \varepsilon_0\omega_p^2 E_z^{\,n} - \omega_0^2 P_z^{\,n} \end{pmatrix}$$

With magnetic field in plasma

$$\frac{d\boldsymbol{J}}{dt} + \nu\boldsymbol{J} + \omega_0^{\,2}\boldsymbol{P} = \varepsilon_0\omega_p^{\,2}\left(\boldsymbol{E} + \frac{\boldsymbol{J}}{en_e}\times\boldsymbol{B}\right) \xrightarrow{\text{FDTD}} \frac{\mathbf{J}^{n+\frac{1}{2}} - \mathbf{J}^{n-\frac{1}{2}}}{\Delta t} + \nu\frac{\mathbf{J}^{n+\frac{1}{2}} + \mathbf{J}^{n-\frac{1}{2}}}{2} + \omega_0^2\mathbf{P}^n = \varepsilon_0\omega_p^2\mathbf{E}^n + \frac{\varepsilon_0\omega_p^2}{en_e}\frac{\mathbf{J}^{n+\frac{1}{2}} + \mathbf{J}^{n-\frac{1}{2}}}{2}\times\mathbf{B}_0$$

$$\boldsymbol{J} = \frac{d\boldsymbol{P}}{dt} \xrightarrow{\text{FDTD}} \frac{\mathbf{P}^{n+1} - \mathbf{P}^n}{\Delta t} = \mathbf{J}^{n+\frac{1}{2}}$$

1. C. Wang et al. JJAP, Vol.63, No.9, 09SP08 (5 pages), Sept. 2024.

FDTD formulation of electromagnetic waves in plasma

Millimeter-wave differential

$$\boldsymbol{H}^{n+\frac{1}{2}} = \boldsymbol{H}^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\nabla \times \boldsymbol{E}^{n}$$

$$H_x^{n+\frac{1}{2}} = H_x^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\left(\frac{\partial E_z^n}{\partial y} - \frac{\partial E_y^n}{\partial z}\right)$$

$$H_y^{n+\frac{1}{2}} = H_y^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\left(\frac{\partial E_x^n}{\partial z} - \frac{\partial E_z^n}{\partial x}\right)$$

$$H_z^{n+\frac{1}{2}} = H_z^{n-\frac{1}{2}} - \frac{\Delta t}{\mu}\left(\frac{\partial E_y^n}{\partial x} - \frac{\partial E_x^n}{\partial y}\right)$$

$$\boldsymbol{E}^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\boldsymbol{E}^{n} + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\nabla \times \boldsymbol{H}^{n+\frac{1}{2}} - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\boldsymbol{J}^{n+\frac{1}{2}}$$

$$E_x^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}E_x^n + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\left(\frac{\partial H_z^{n+\frac{1}{2}}}{\partial y} - \frac{\partial H_y^{n+\frac{1}{2}}}{\partial z}\right) - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}J_x^{n+\frac{1}{2}}$$

$$E_y^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}E_y^n + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\left(\frac{\partial H_x^{n+\frac{1}{2}}}{\partial z} - \frac{\partial H_z^{n+\frac{1}{2}}}{\partial x}\right) - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}J_y^{n+\frac{1}{2}}$$

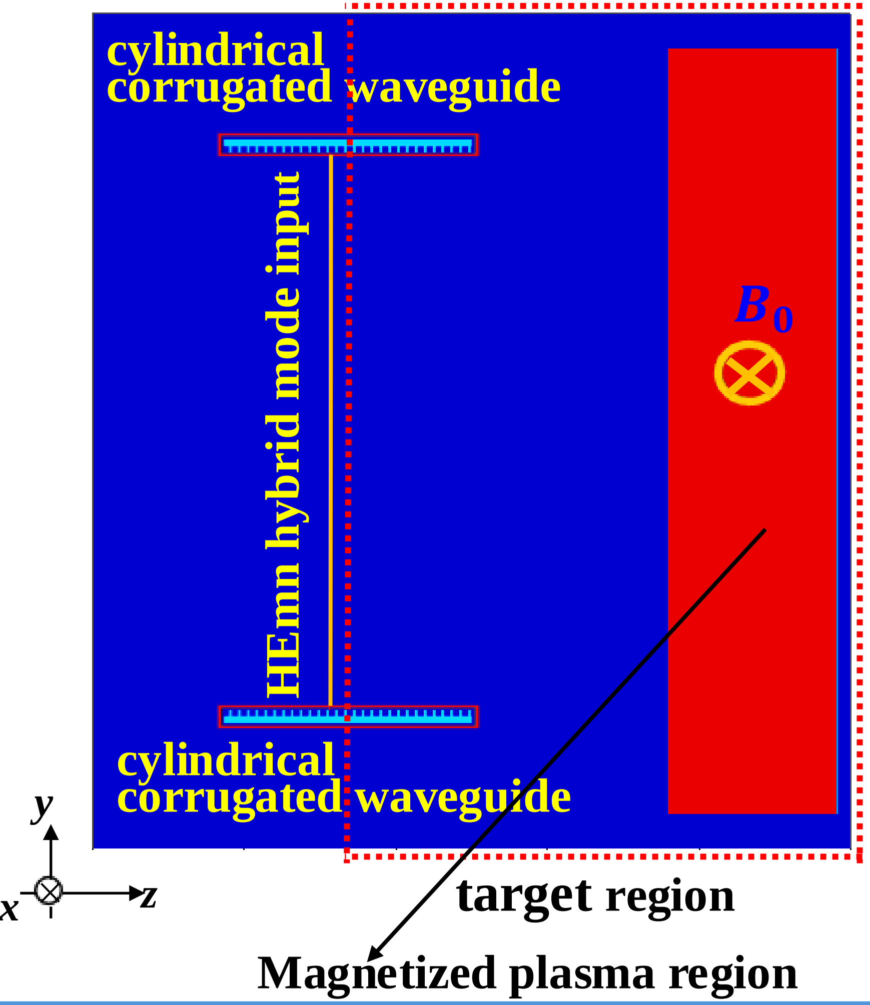
$$E_z^{n+1} = \frac{\frac{\varepsilon}{\Delta t} - \frac{\sigma}{2}}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}E_z^n + \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}\left(\frac{\partial H_y^{n+\frac{1}{2}}}{\partial x} - \frac{\partial H_x^{n+\frac{1}{2}}}{\partial y}\right) - \frac{1}{\frac{\varepsilon}{\Delta t} + \frac{\sigma}{2}}J_z^{n+\frac{1}{2}}$$

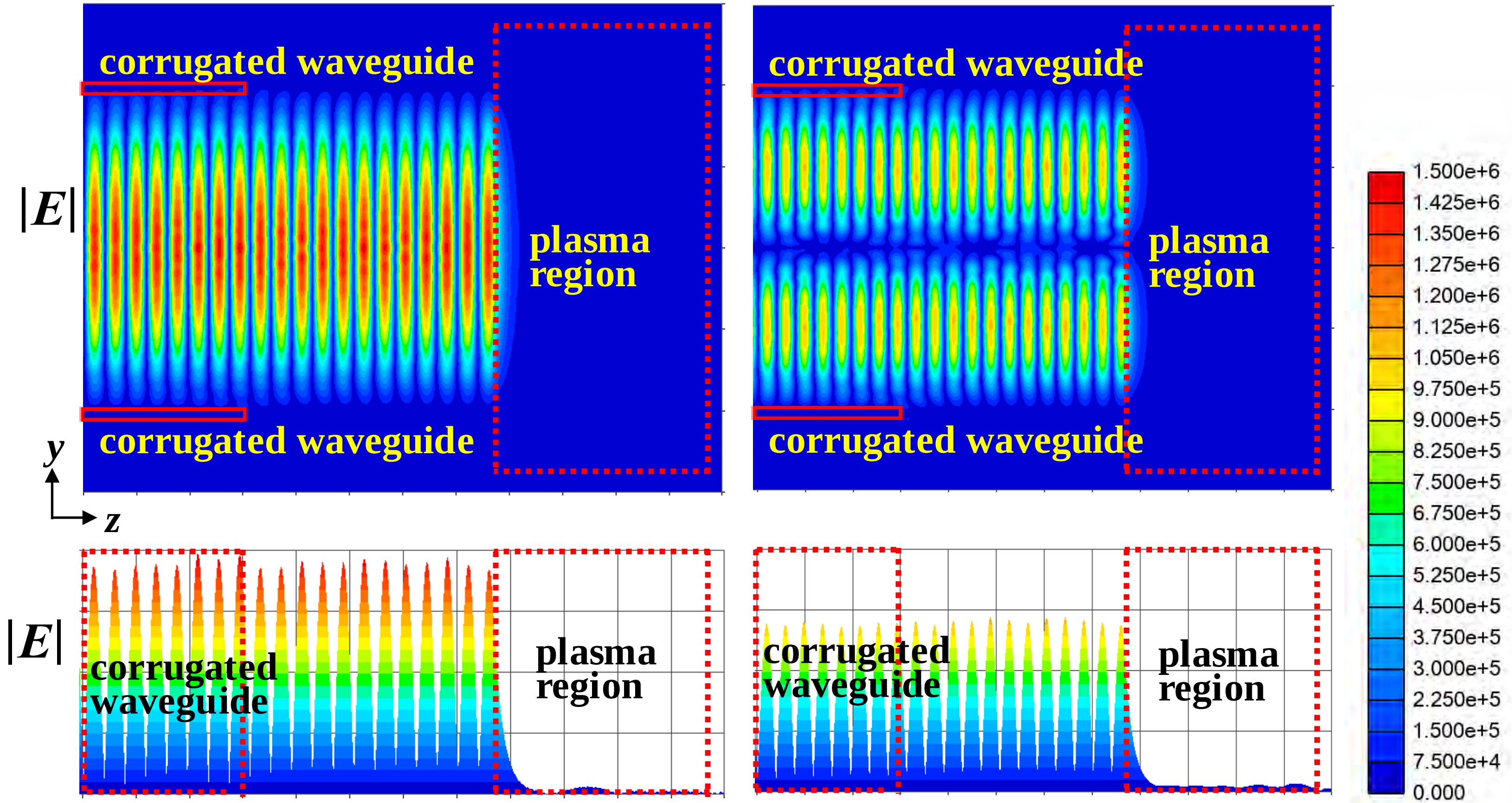
Drude-Lorentz model matrix

$$\begin{pmatrix} \frac{1}{\Delta t} + \frac{\nu}{2} & -\frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{z0}}{2} & \frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{y0}}{2} \\ \frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{y0}}{2} & \frac{1}{\Delta t} + \frac{\nu}{2} & -\frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{z0}}{2} \\ -\frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{z0}}{2} & \frac{\varepsilon_0\omega_p^2}{en_e}\frac{B_{y0}}{2} & \frac{1}{\Delta t} + \frac{\nu}{2} \end{pmatrix} \begin{pmatrix} J_x^{n+\frac{1}{2}} \\ J_y^{n+\frac{1}{2}} \\ J_z^{n+\frac{1}{2}} \end{pmatrix} = \begin{pmatrix} \left(-\frac{1}{\Delta t} + \frac{\nu}{2}\right)J_x^{n-\frac{1}{2}} + \frac{\varepsilon_0\omega_p^2}{2en_e}\left(J_y^{n-\frac{1}{2}}B_{z0} - J_z^{n-\frac{1}{2}}B_{y0}\right) + \varepsilon_0\omega_p^2E_x^n - \omega_0^2P_x^n \\ \left(-\frac{1}{\Delta t} + \frac{\nu}{2}\right)J_y^{n-\frac{1}{2}} + \frac{\varepsilon_0\omega_p^2}{2en_e}\left(J_z^{n-\frac{1}{2}}B_{x0} - J_x^{n-\frac{1}{2}}B_{z0}\right) + \varepsilon_0\omega_p^2E_y^n - \omega_0^2P_y^n \\ \left(-\frac{1}{\Delta t} + \frac{\nu}{2}\right)J_z^{n-\frac{1}{2}} + \frac{\varepsilon_0\omega_p^2}{2en_e}\left(J_x^{n-\frac{1}{2}}B_{y0} - J_y^{n-\frac{1}{2}}B_{x0}\right) + \varepsilon_0\omega_p^2E_z^n - \omega_0^2P_z^n \end{pmatrix}$$

Numerical results

f=77 GHz, $B_{0\,x}$ = 2 T, n_e = 1.103 x 10^{20} , 800*800*600





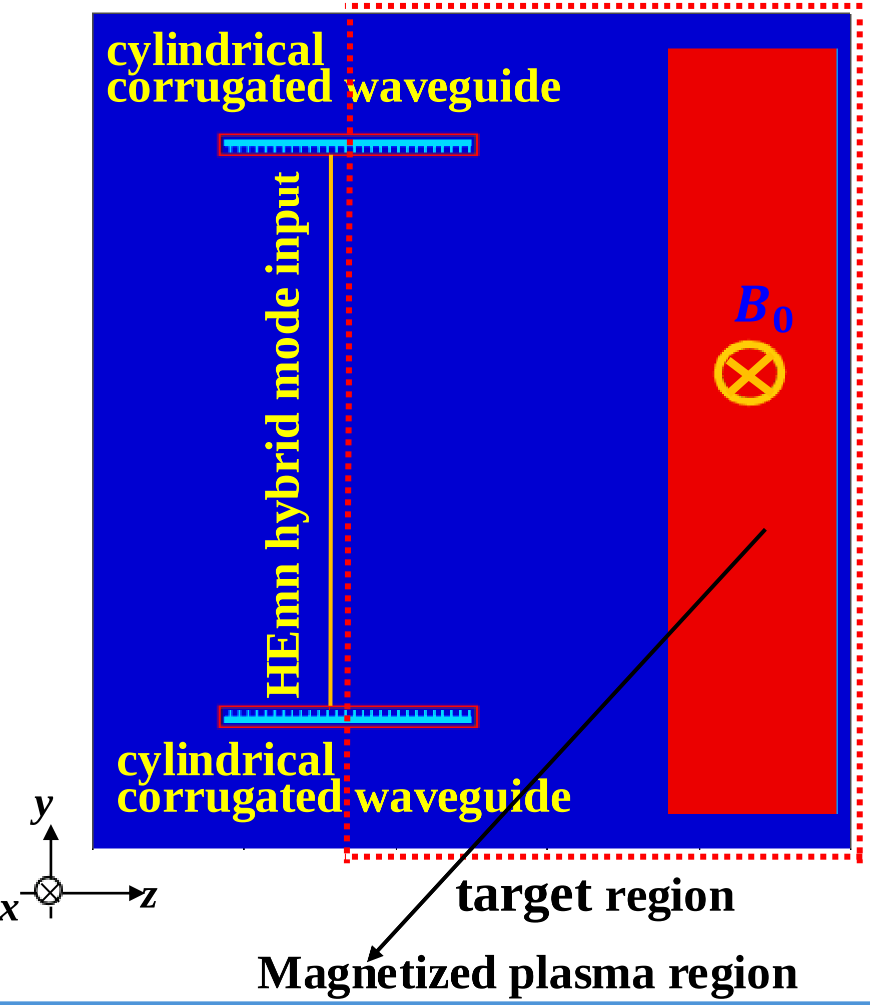
(a) l=0 (b) l=1

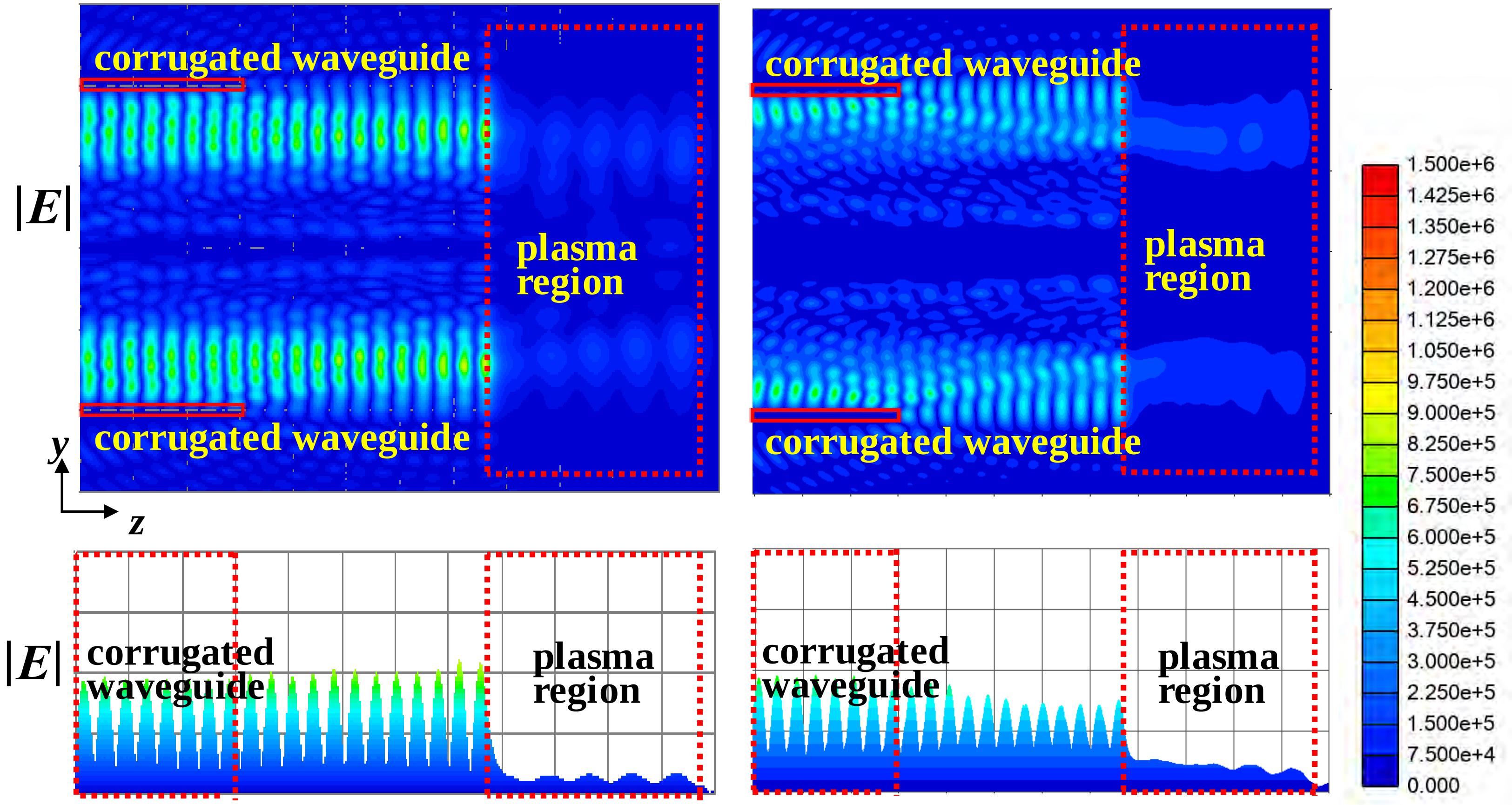
Distribution of absolute value of electric field intensity

1. C. Wang et al. JJAP, Vol.63, No.9, 09SP08 (5 pages), Sept. 2024.

Numerical results

f=77 GHz, $B_{0\,x}$ = 2 T, n_e = 1.103 x 10^{20} , 800*800*600





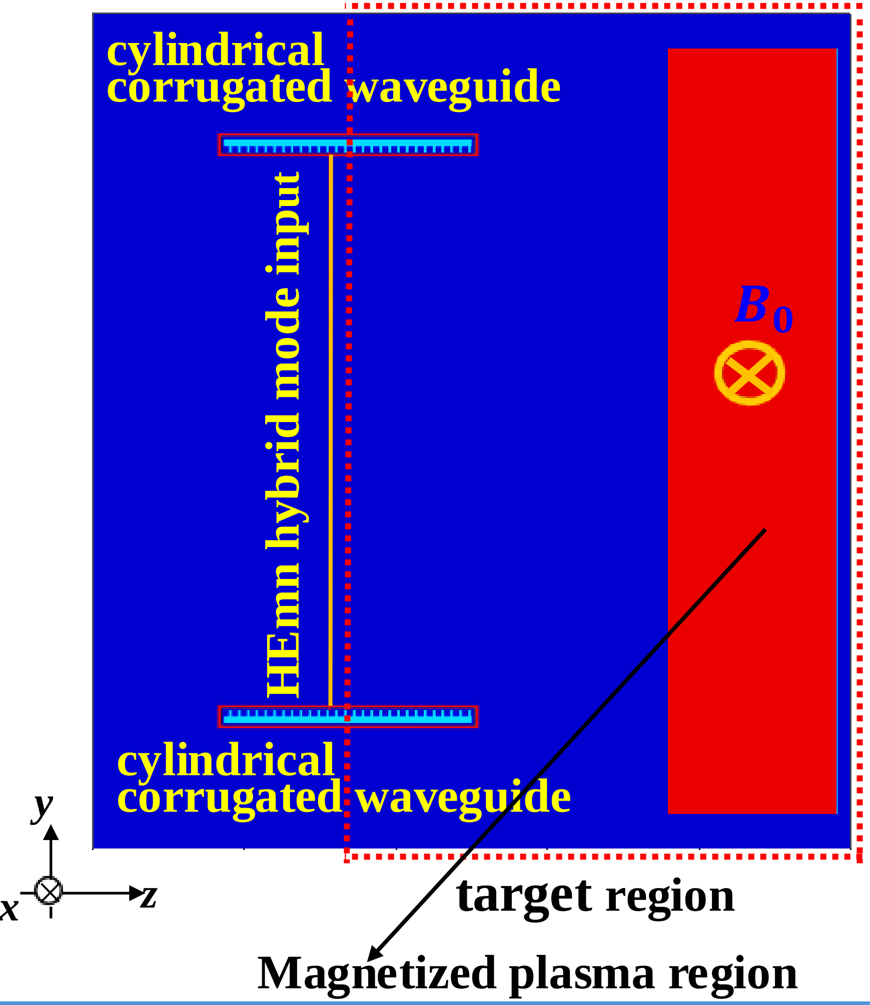
(a) l=5 (b) l=10

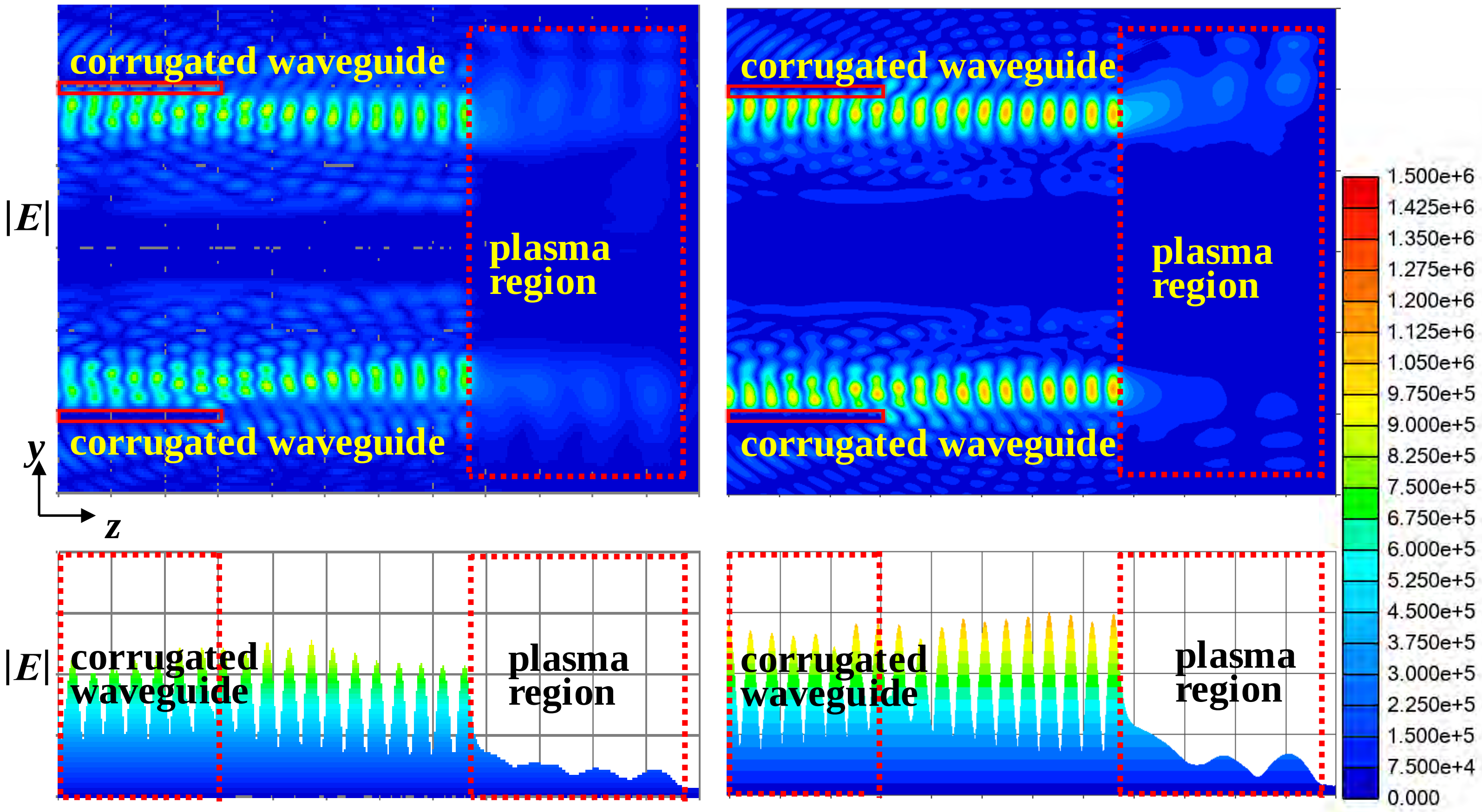
Distribution of absolute value of electric field intensity

1. C. Wang et al. JJAP, Vol.63, No.9, 09SP08 (5 pages), Sept. 2024.

Numerical results

f=77 GHz, $B_{0\,x}$ = 2 T, n_e = 1.103 x 10^{20} , 800*800*600





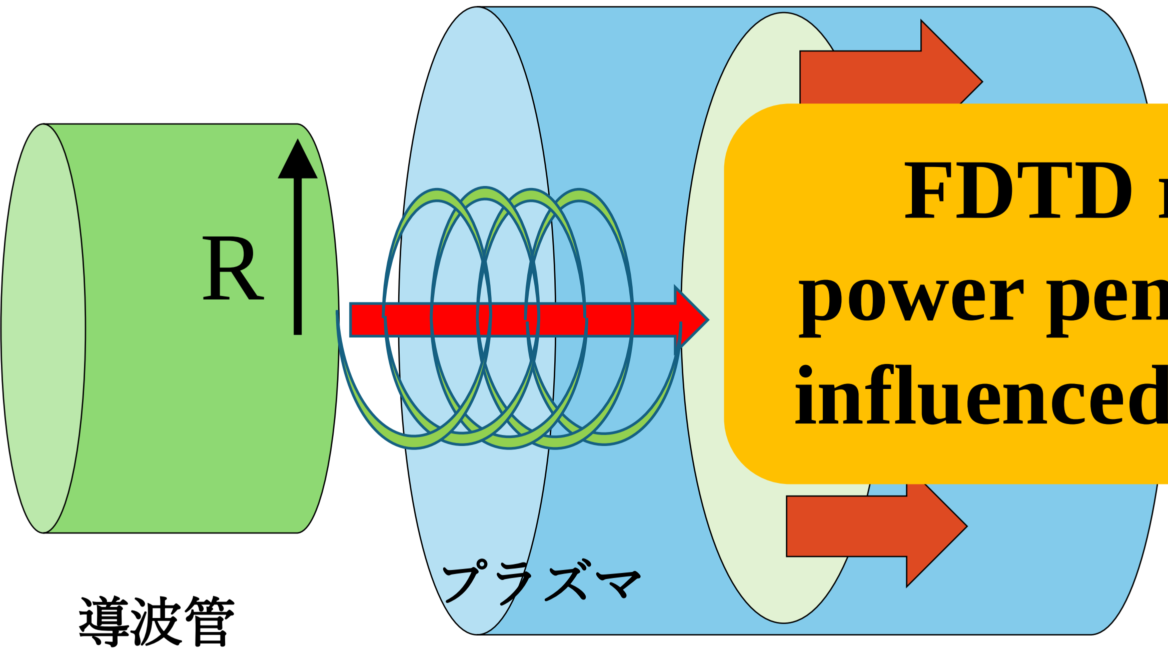
(a) l=15 (b) l=20

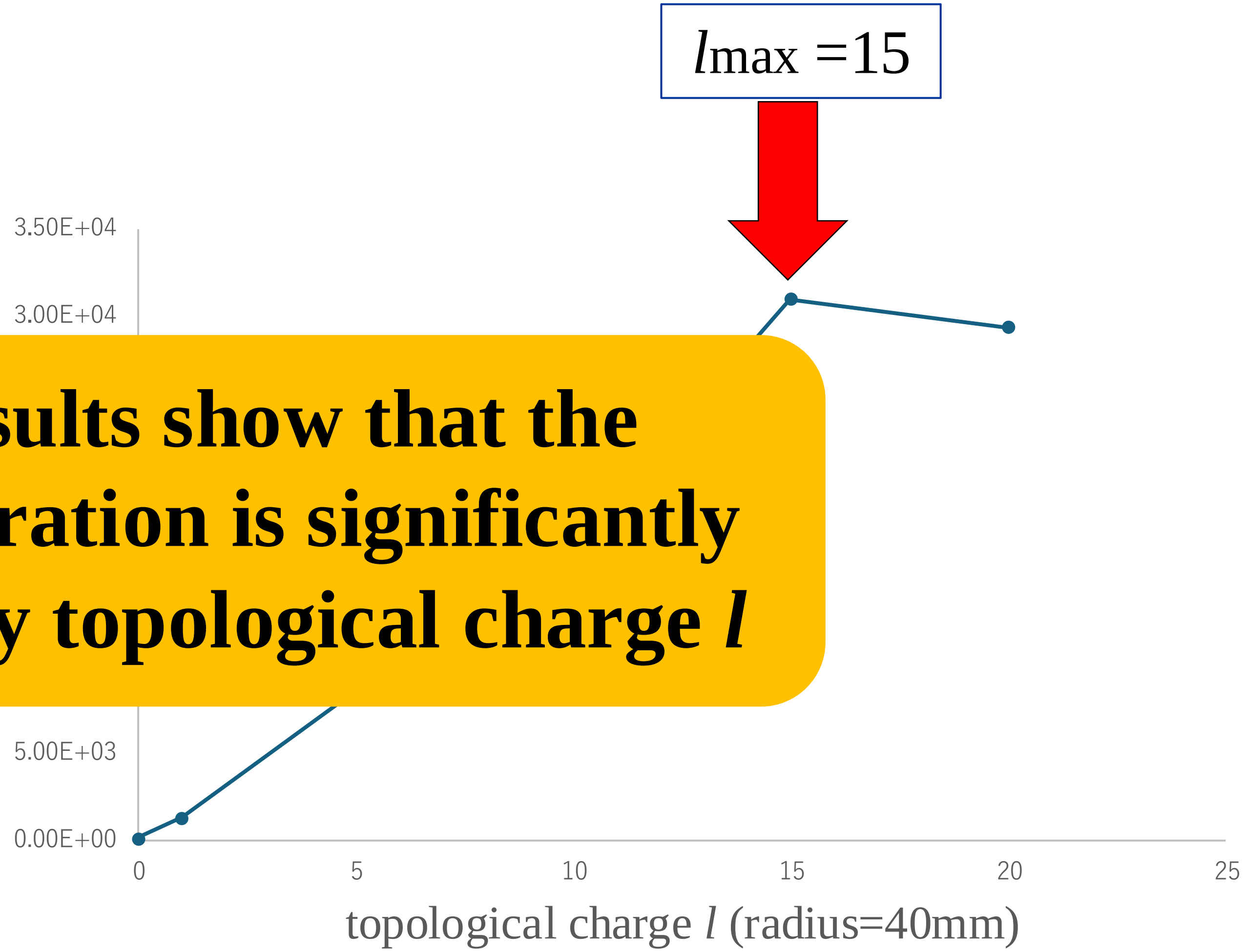
Distribution of absolute value of electric field intensity

1. C. Wang et al. JJAP, Vol.63, No.9, 09SP08 (5 pages), Sept. 2024.

The dependence of penetrated power in plasma

P:断面のエネルギー流の積分





FDTD results show that the power penetration is significantly influenced by topological charge *l*

Summary

- Propagation characteristic of the millimeter-wave vortex of the cylindrical corrugated waveguide hybrid mode in the magnetized plasma is discussed by using FDTD method.

- It is confirmed that the hybrid mode for millimeter-wave vortex field can propagate in the magnetized plasma region in which the normal plane wave is in cut-off condition.

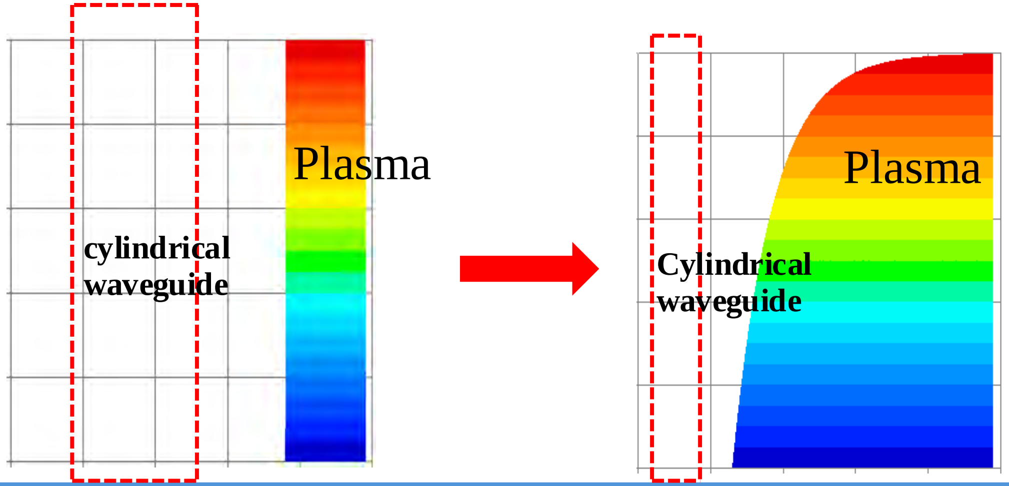
- The dependence of propagation of the hybrid mode millimeter-wave vortex in the magnetized plasma on topological charge l is investigated.

- The simulation results demonstrate that the penetrated power of millimeter-wave vortices in magnetized plasma is significantly influenced by the topological charge .

Future Plan

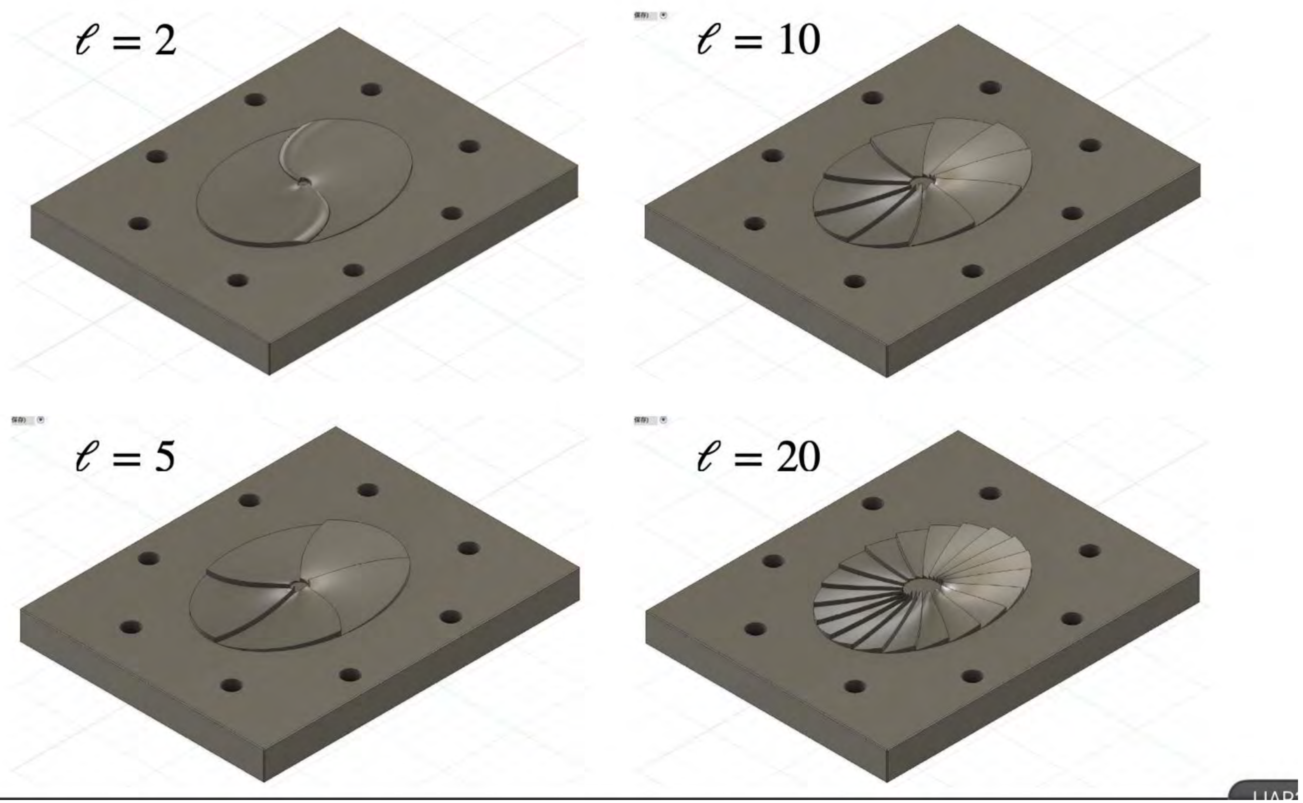
1. Update the plasma parameters in the model to reflect the actual CHD/CHD-U device conditions.
2. Non-uniform distribution of plasma density will be considered.
   ・Thermal plasma will be considered and the absoption power will be investigated by using PIC (Particle in Cell) method.

3. To Construct a new ECE system in the CHD/CHD-U device that enables observation of more information of plasma



Appendix

The generation of OV in experimentals (by Kubo.S sensei)



Phase conversion mirror